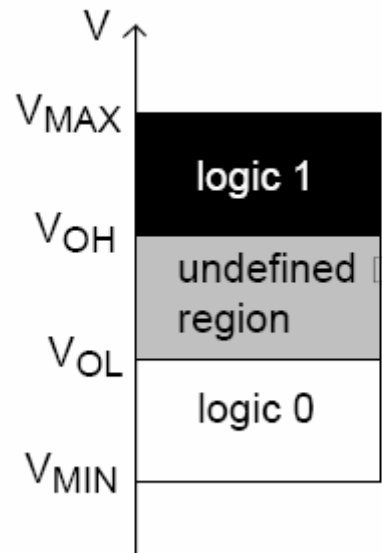


Lect. 20: Inverters (S&S 10.1-2)

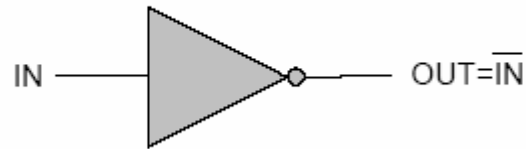
Digital Electronics



How to implement logic gates?

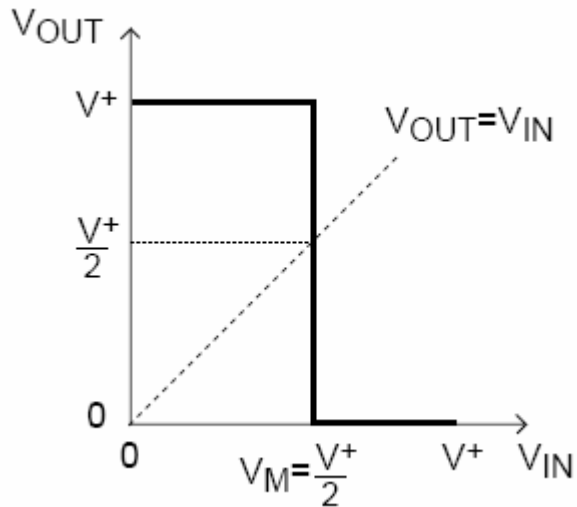
→ Inverters are the basic building block!

Lect. 20: Inverters

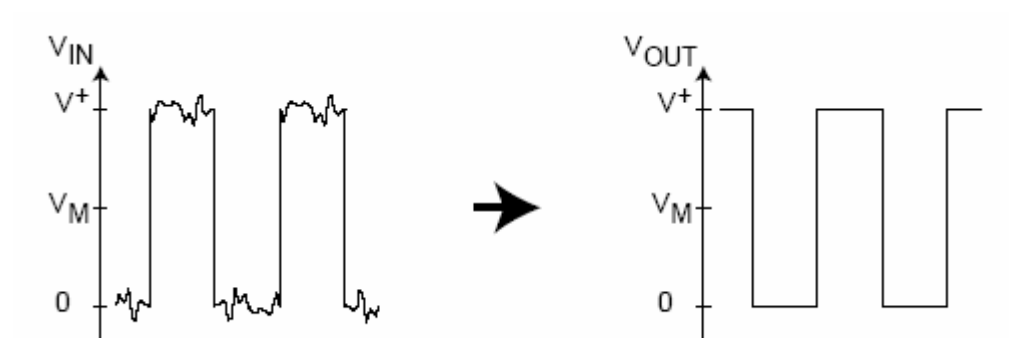


IN	OUT
0	1
1	0

Ideally,

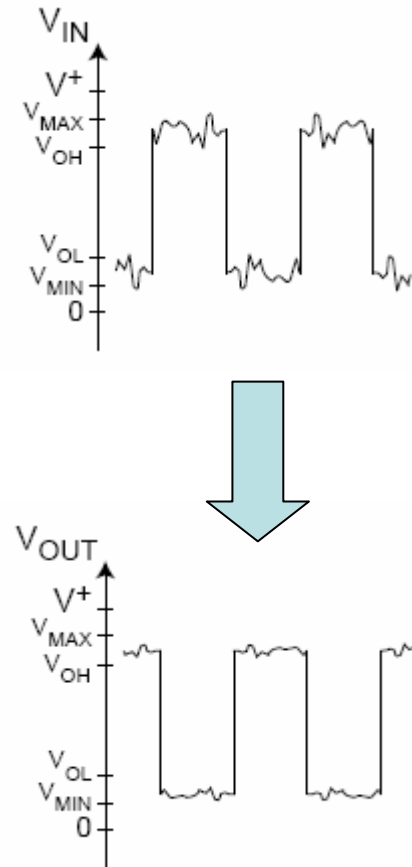
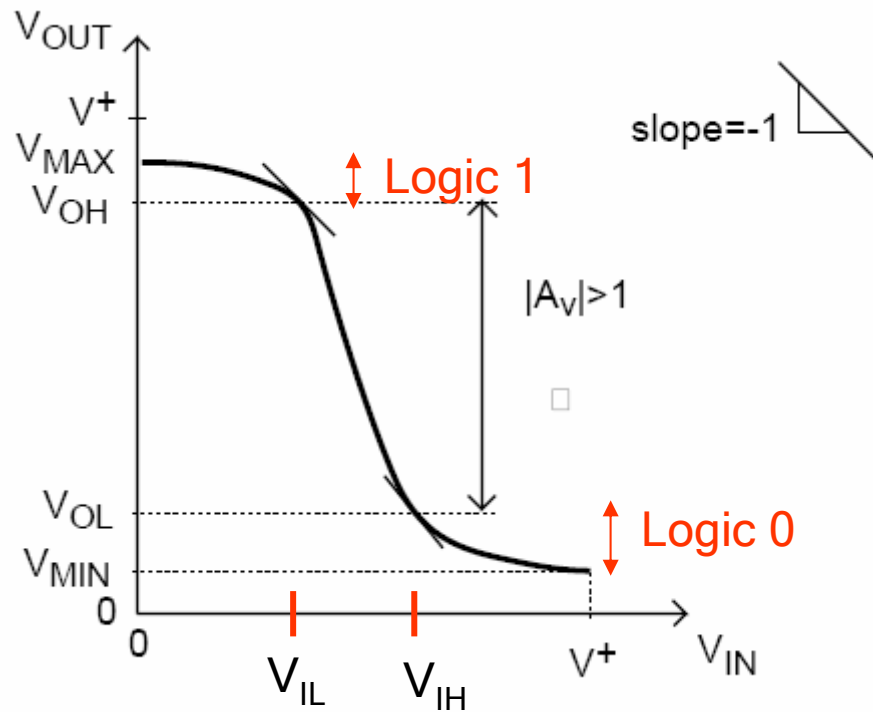


Signal can be regenerated



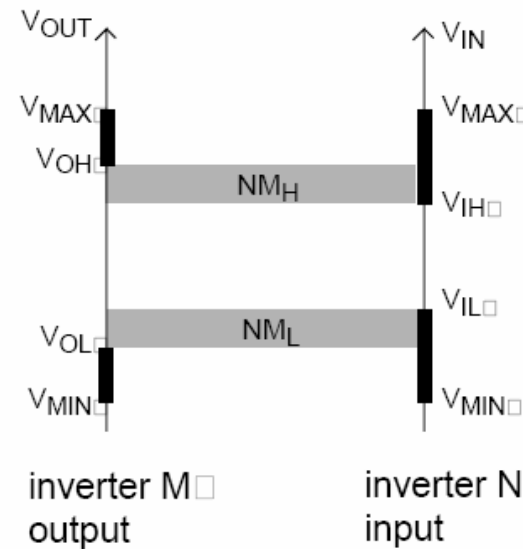
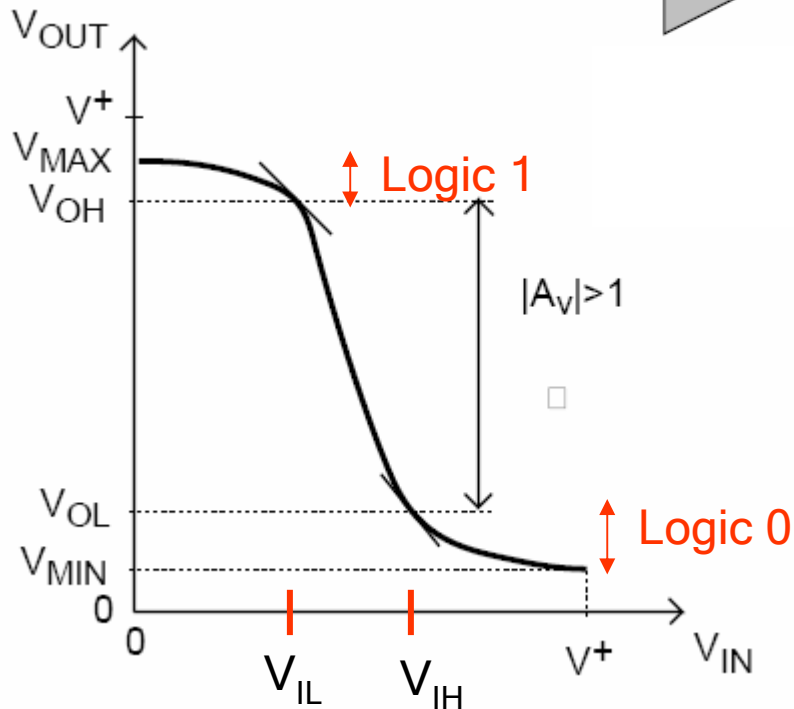
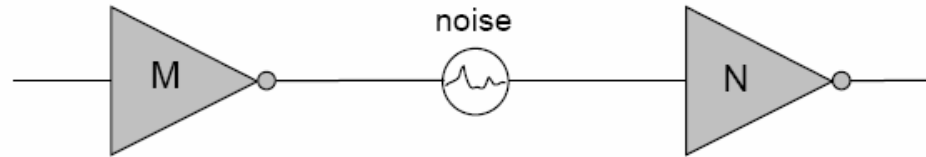
Lect. 20: Inverters

Real inverters



Lect. 20: Inverters

Noise Margin



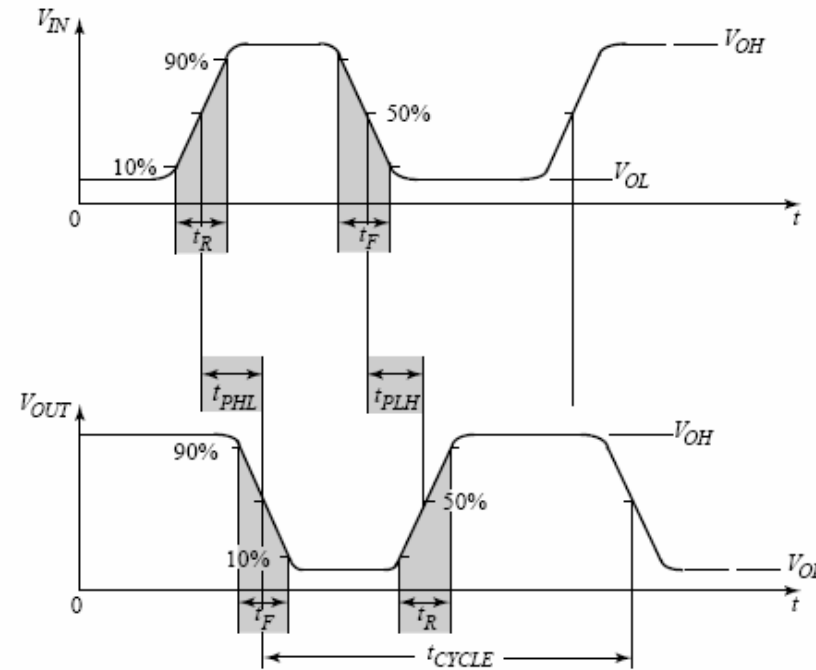
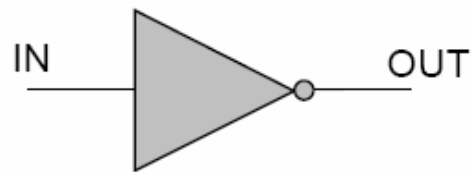
$$NM_H = V_{OH} - V_{IH}$$

$$NM_L = V_{IL} - V_{OL}$$

Larger noise margin \rightarrow smaller V_{IH} , larger V_{IL} \rightarrow Larger $|A_V|$

Lect. 20: Inverters

Transient Characteristics

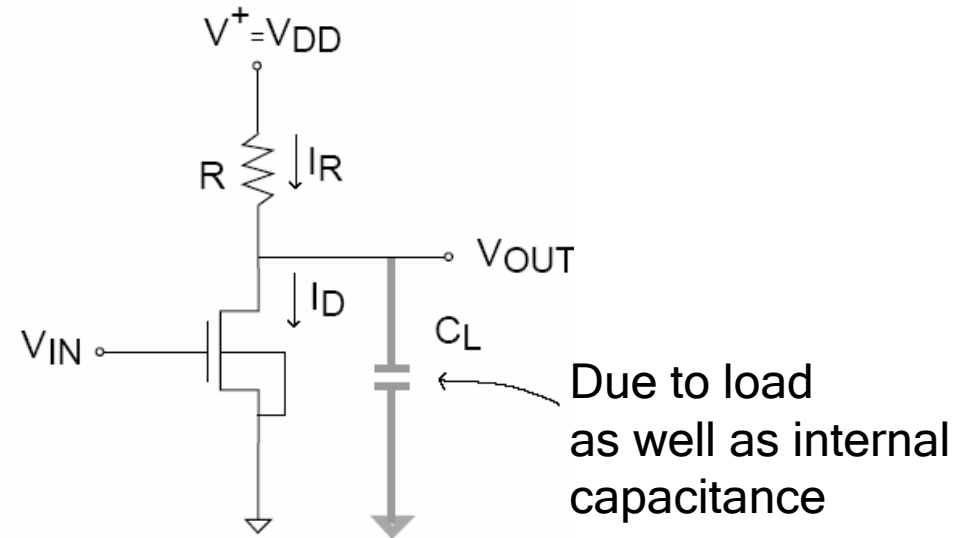
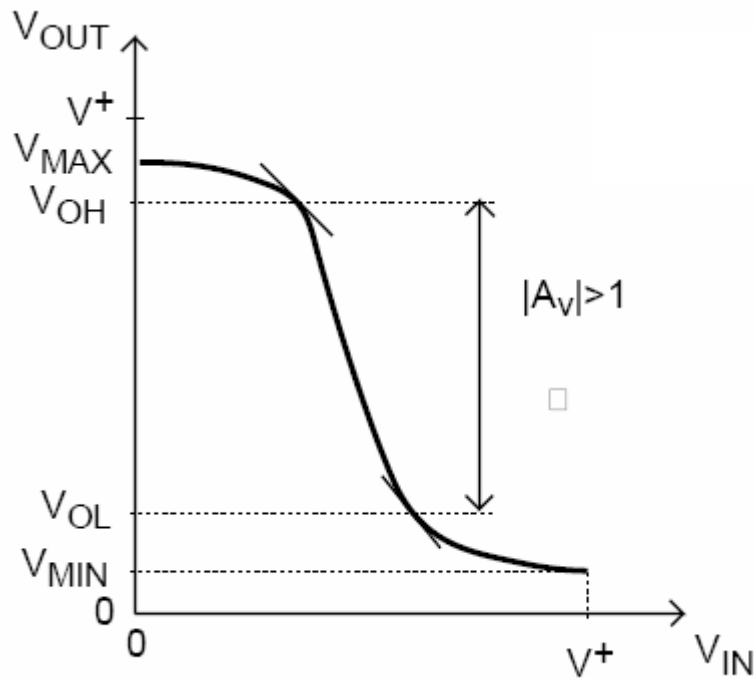


Propagation Delay:

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH})$$

Lect. 20: Inverters

How to implement an inverter? CS amplifier configuration



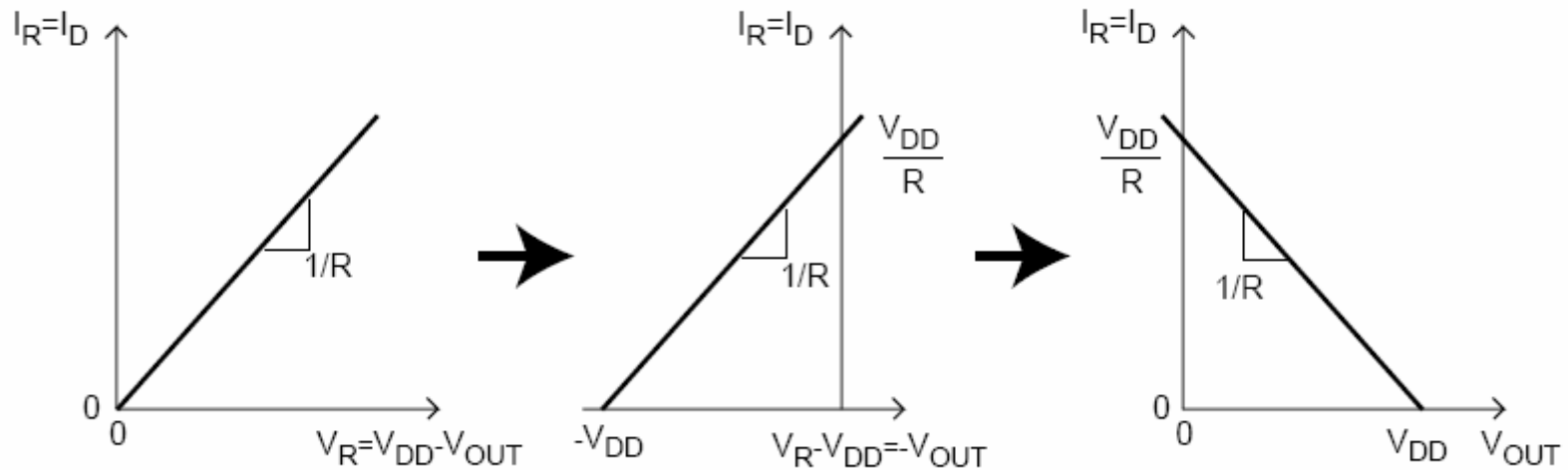
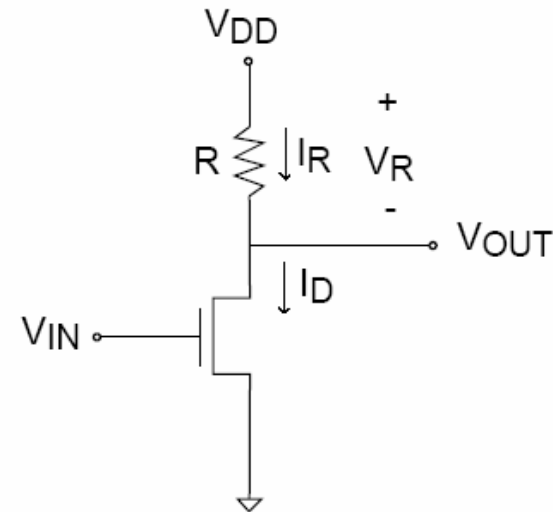
$V_{IN} < V_T$, MOSFET OFF $\Rightarrow V_{OUT} = V_{DD}$

$V_{IN} > V_T$, MOSFET ON $\Rightarrow V_{OUT}$ small

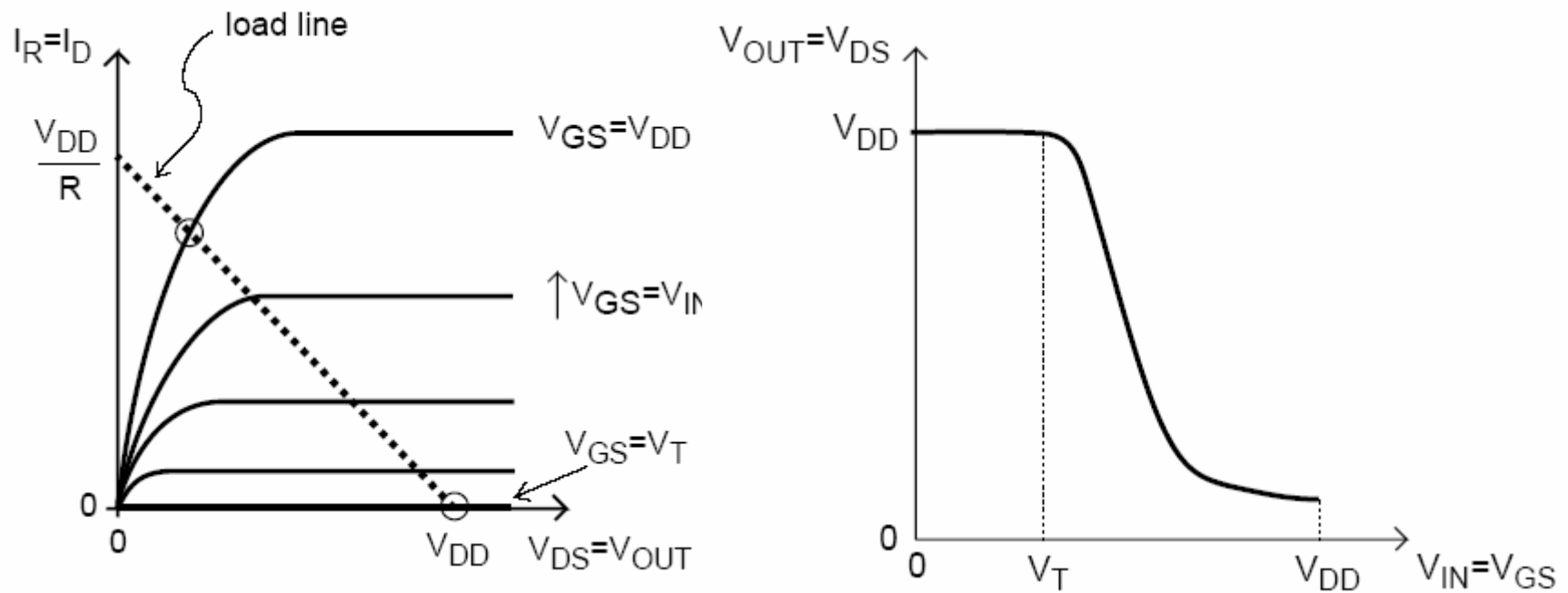
Lect. 20: Inverters

NMOS inverter with R:

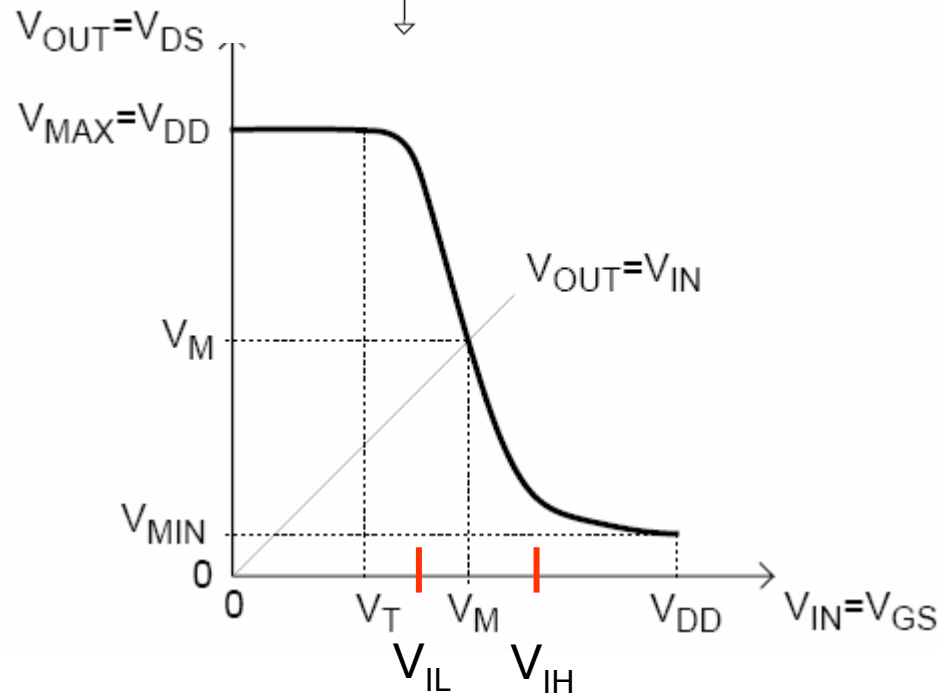
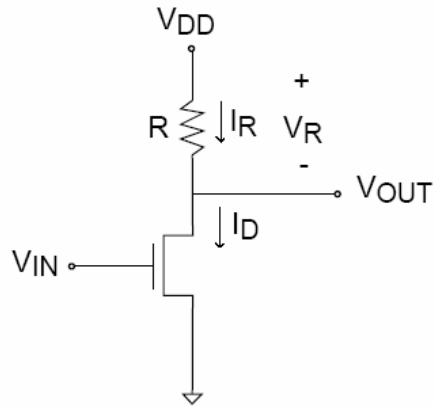
Transfer Function using load-line analysis



Lect. 20: Inverters



Lect. 20: Inverters



For V_{MIN} ,

$$I_D = \frac{W}{L} \mu_n C_{ox} \left(V_{DD} - \frac{V_{MIN}}{2} - V_T \right) V_{MIN}$$

$$= I_R = \frac{V_{DD} - V_{MIN}}{R}$$

For V_M ,

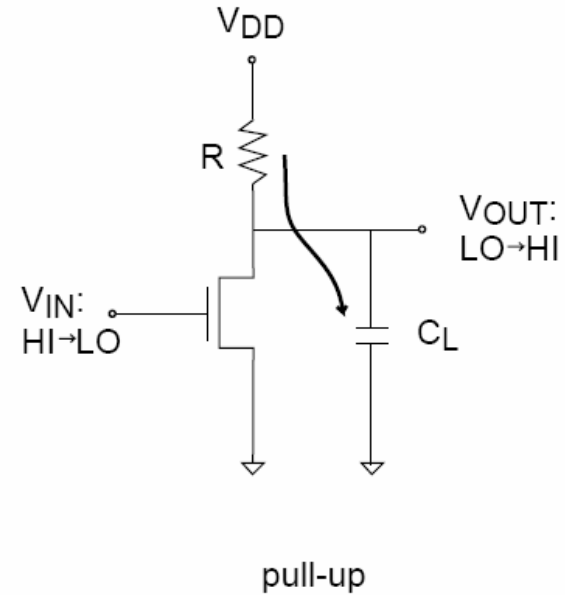
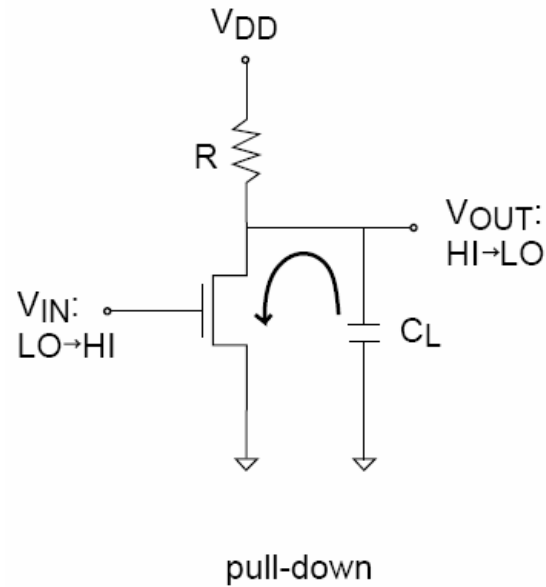
$$I_D = \frac{W}{2L} \mu_n C_{ox} (V_M - V_T)^2$$

$$= I_R = \frac{V_{DD} - V_M}{R}$$

Larger Noise Margin \rightarrow Larger $|A_v|$
 \rightarrow Large g_m and R

Lect. 20: Inverters

Dynamics



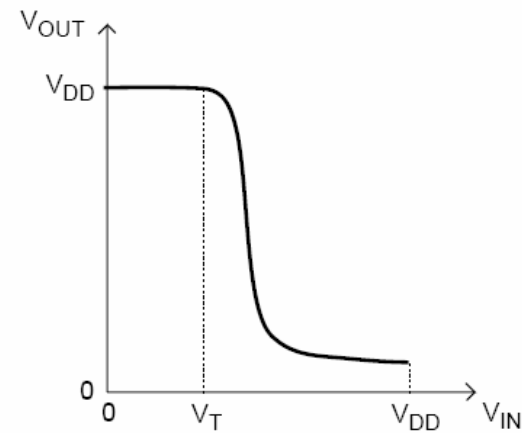
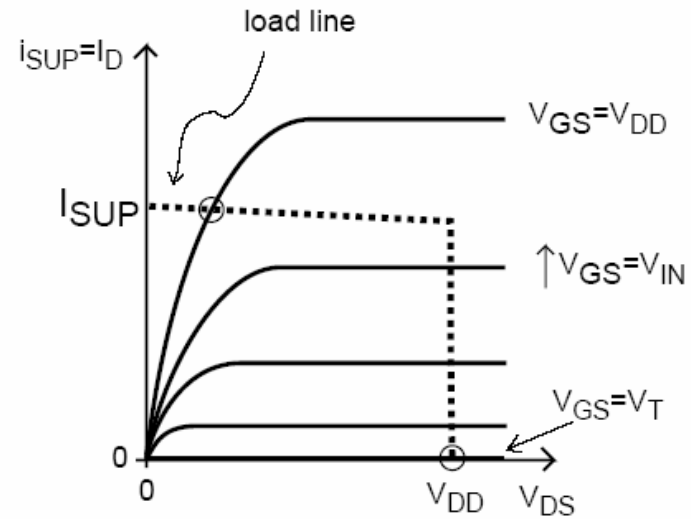
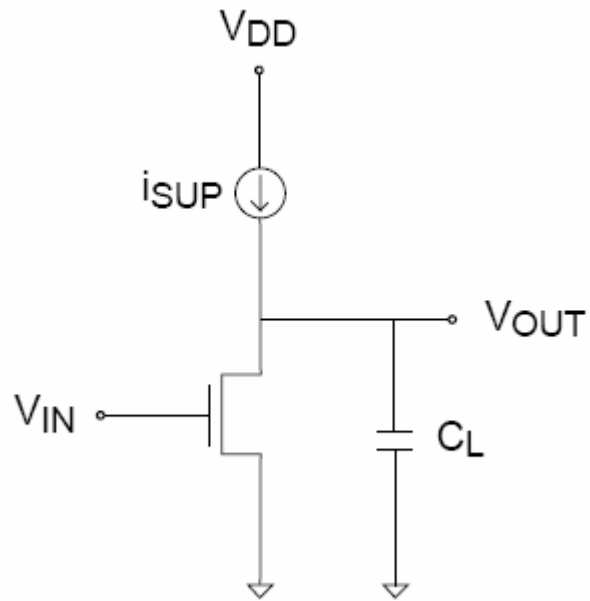
For faster operation: Small RC

For large noise margin: Large $|A_v|$ → Large R

→ Better way of implementing inverter?

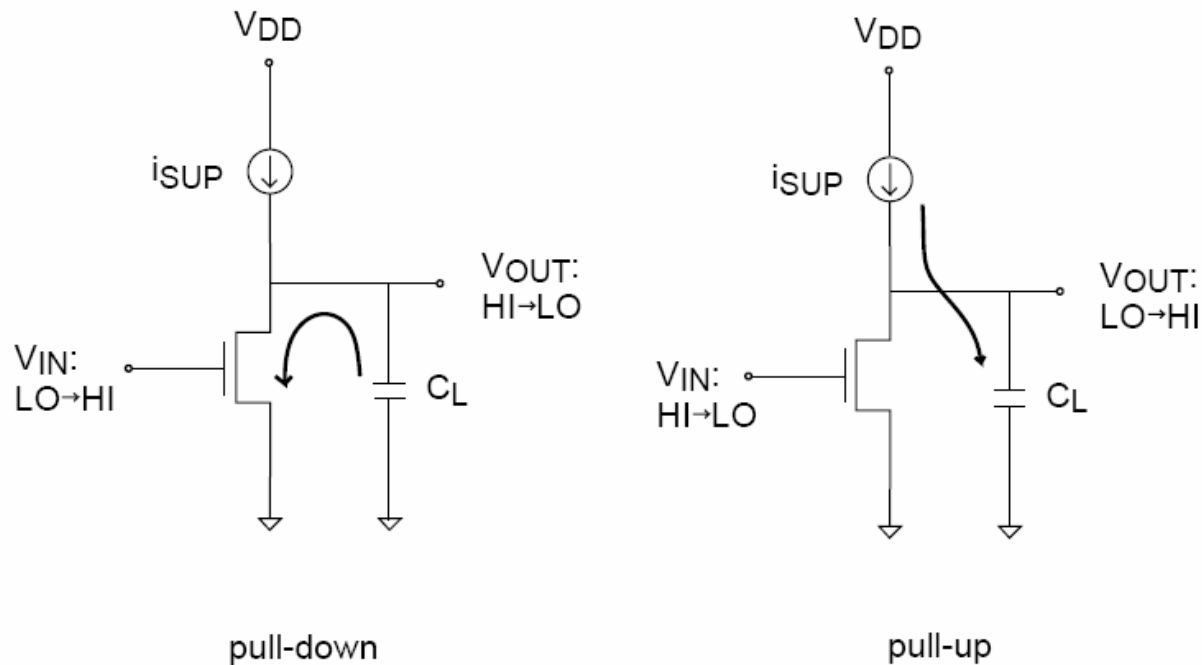
Lect. 20: Inverters

Use current source instead of R



Lect. 20: Inverters

Dynamics:

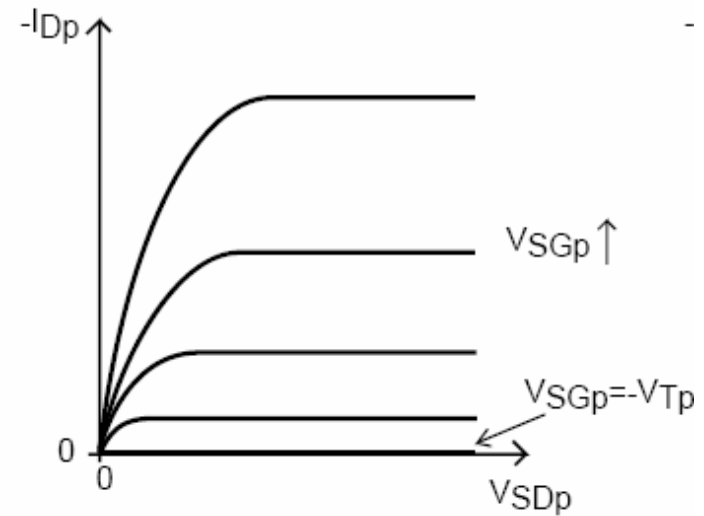
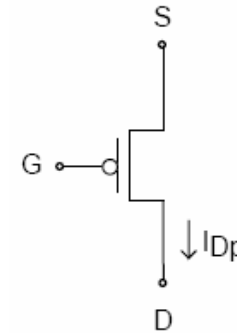
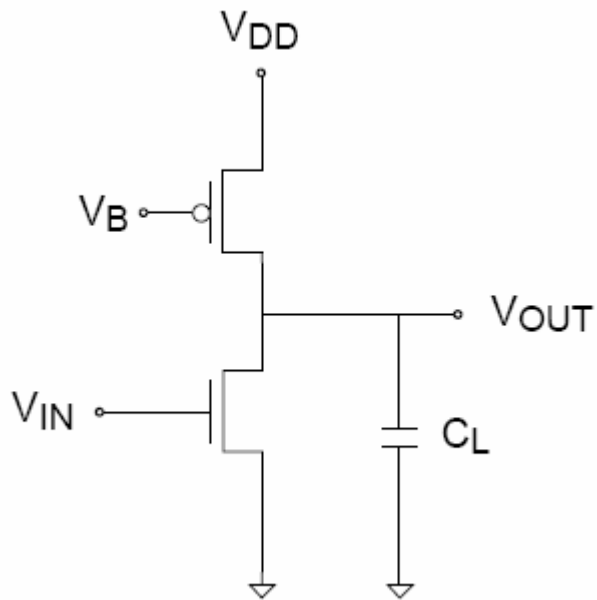


Faster pull-up because capacitor charged at constant current.

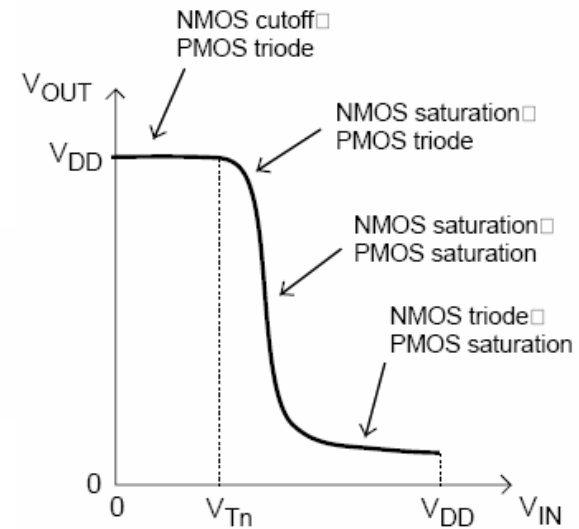
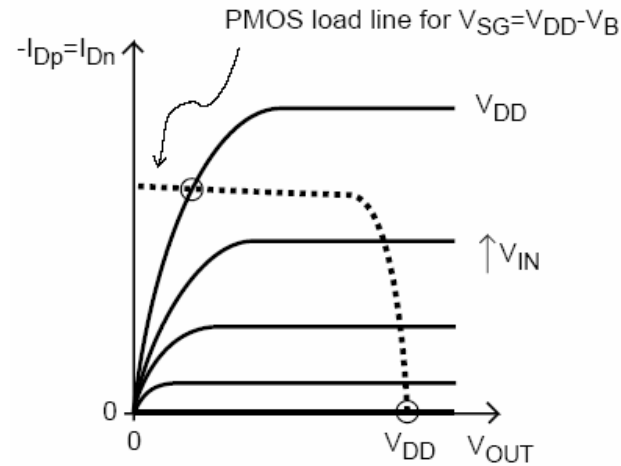
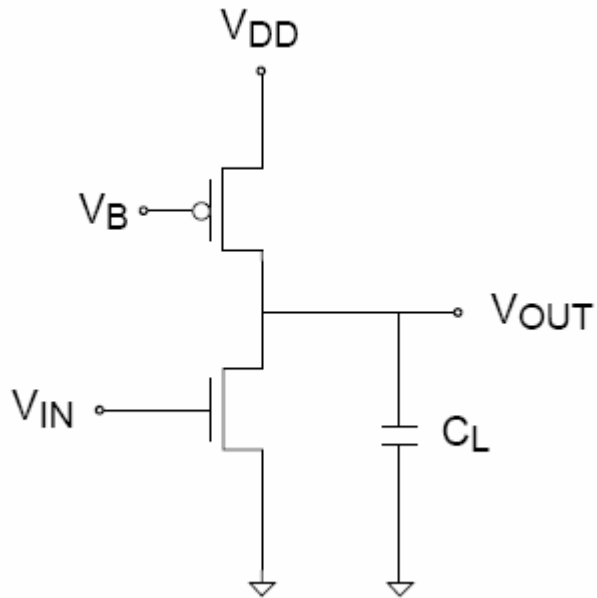
Lect. 20: Inverters

Simplest current source?

Use PMOS



Lect. 20: Inverters

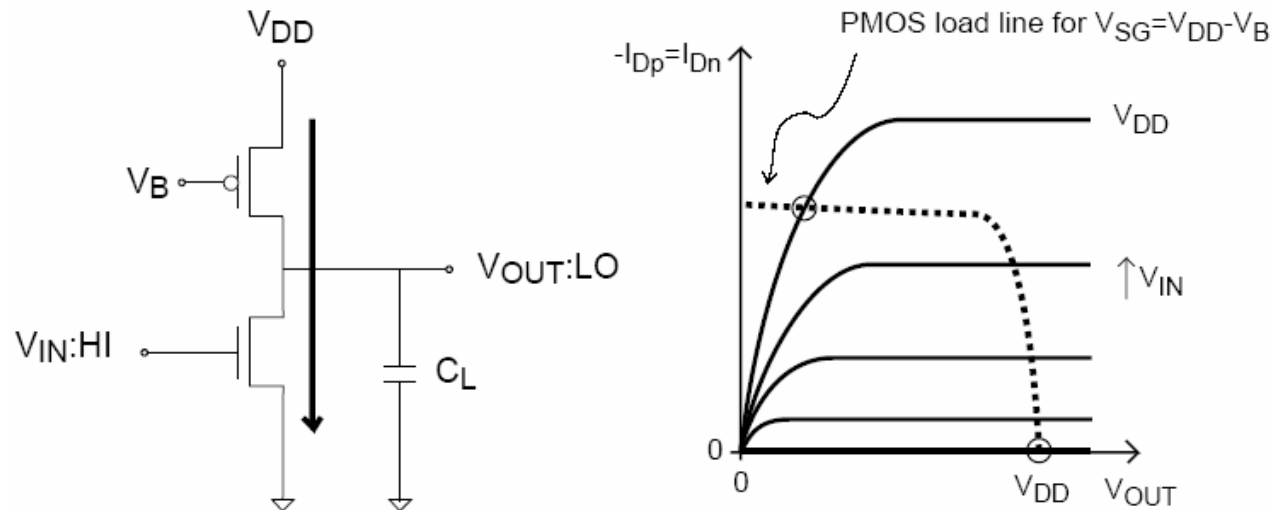


$$A_v = -g_{mn}(r_{on} // r_{op})$$

Lect. 20: Inverters

NMOS inverter with current-source pull-up allows fast switching with high noise margins.

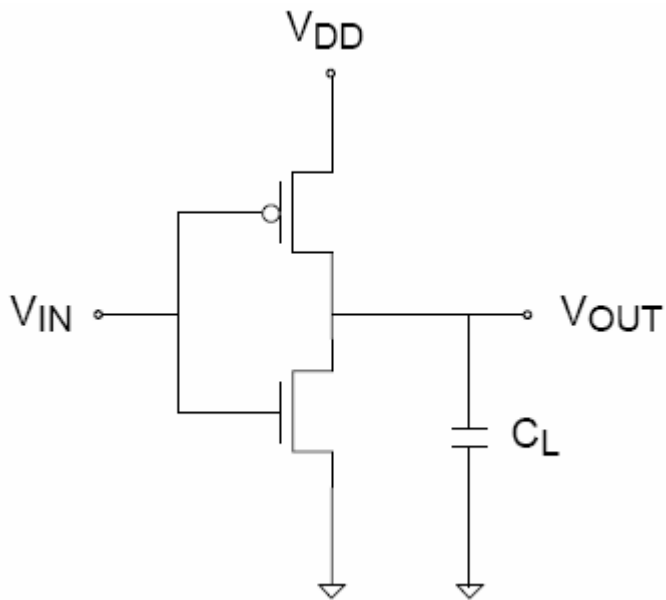
But... when $V_{IN} = V_{DD}$, there is a direct current path between supply and ground



Static Power consumption! → How can we shut it off when input is high?

Lect. 20: Inverters

Complementary MOS (CMOS) Inverter



- $V_{IN} = 0 \Rightarrow V_{OUT} = V_{DD}$

$$V_{GSn} = 0 < V_{Tn} \Rightarrow \text{NMOS OFF}$$

$$V_{SGp} = V_{DD} > -V_{Tp} \Rightarrow \text{PMOS ON}$$

- $V_{IN} = V_{DD} \Rightarrow V_{OUT} = 0$

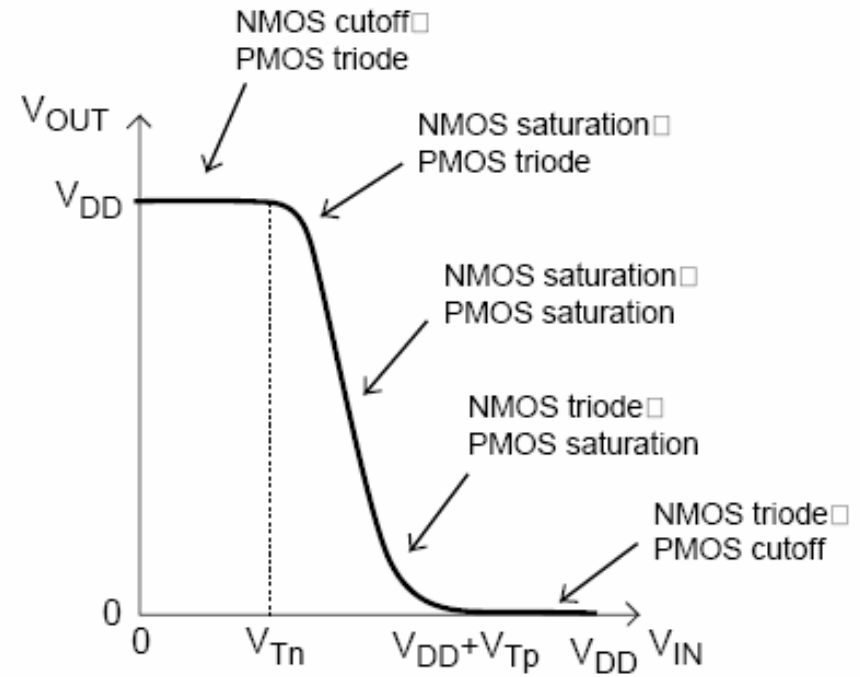
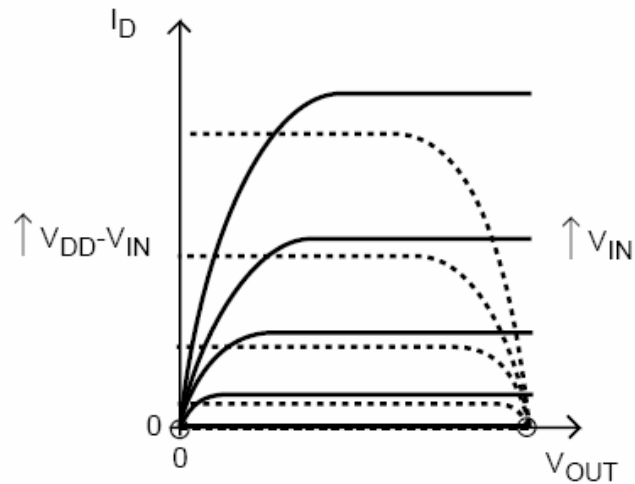
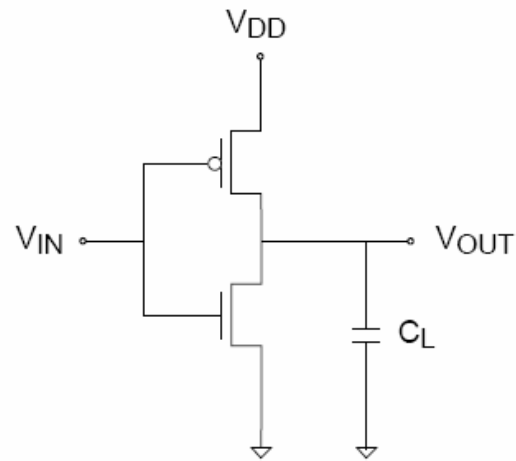
$$V_{GSn} = V_{DD} > V_{Tn} \Rightarrow \text{NMOS ON}$$

$$V_{SGp} = 0 < -V_{Tp} \Rightarrow \text{PMOS OFF}$$

No power consumption while idling in any logic state.

➔ The most popular building block for today's digital electronics!

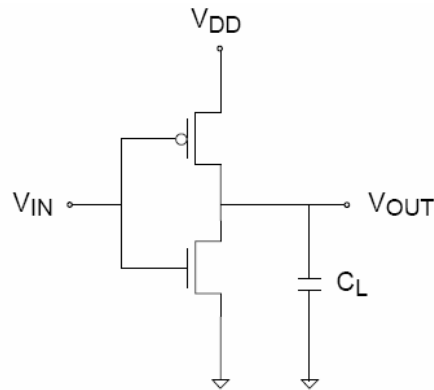
Lect. 20: Inverters



Rail-to-Rail logic

Lect. 20: Inverters

Estimation of important parameters for CMOS inverter: V_M , $A_V(V_M)$, NM_L , NM_H



For V_M ($V_{IN} = V_{OUT} = V_M$)

$$\frac{1}{2} \frac{W_n}{L_n} \mu_n C_{ox} (V_M - V_{Tn})^2 = \frac{1}{2} \frac{W_p}{L_p} \mu_p C_{ox} (V_{DD} - V_M + V_{Tp})^2$$

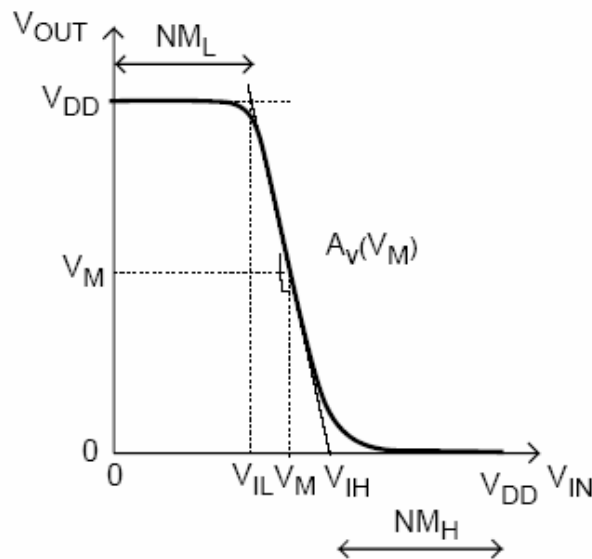
Using $k_n = \frac{W_n}{L_n} \mu_n C_{ox}$, $k_p = \frac{W_p}{L_p} \mu_p C_{ox}$

$$V_M = \frac{V_{Tn} + \sqrt{\frac{k_p}{k_n}} (V_{DD} + V_{Tp})}{1 + \sqrt{\frac{k_p}{k_n}}}$$

V_M can be engineered with controlling k_p/k_n

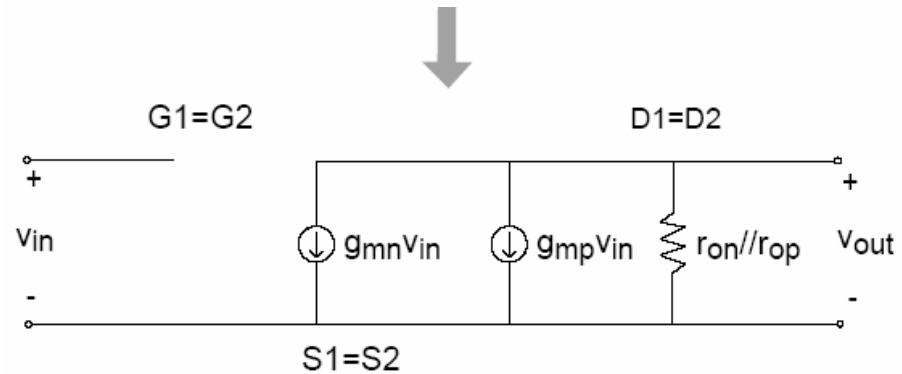
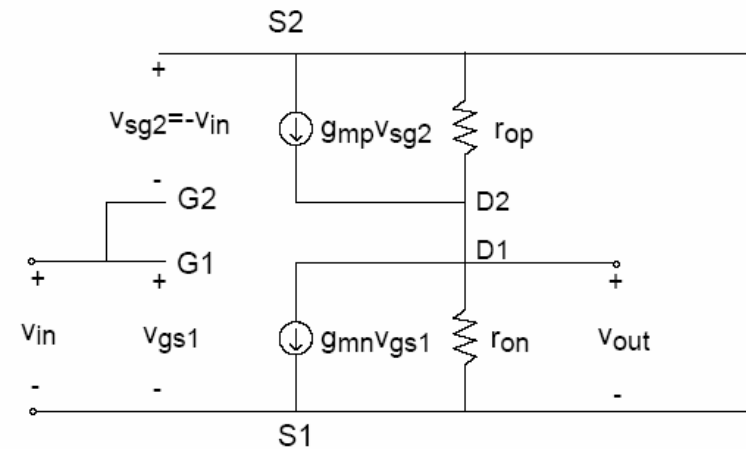
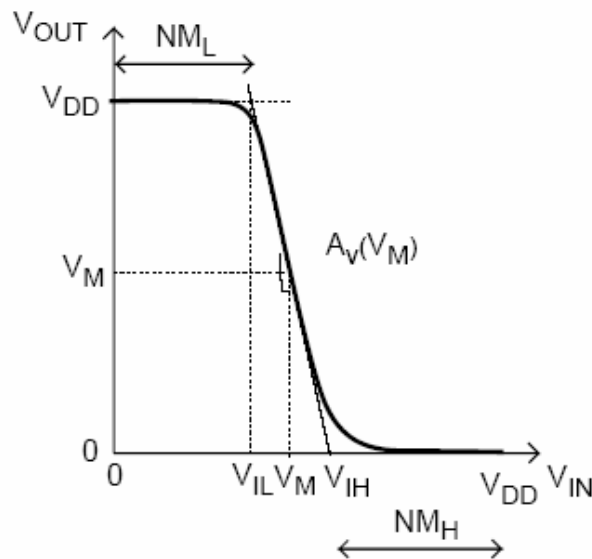
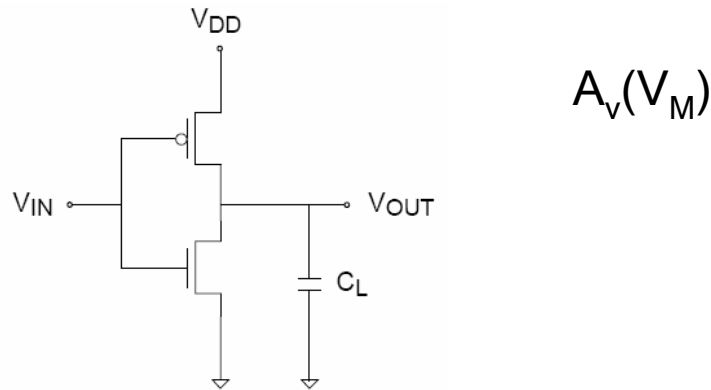
Assuming $V_{Tn} = -V_{Tp}$ and $k_p = k_n$, $V_M = V_{DD}/2$

$$(W_p \approx 3W_n)$$



Lect. 20: Inverters

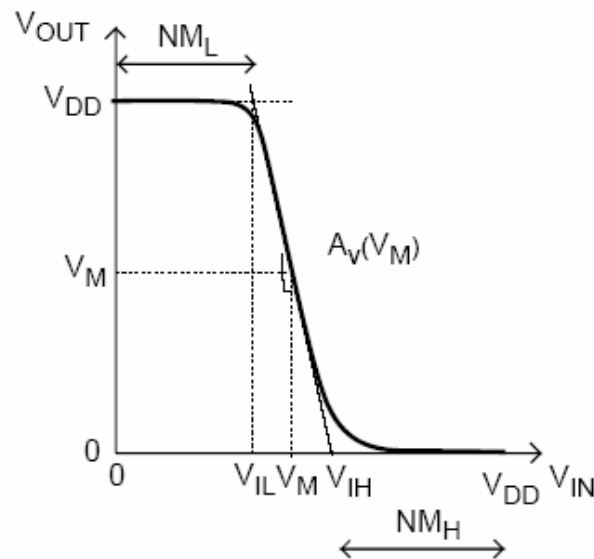
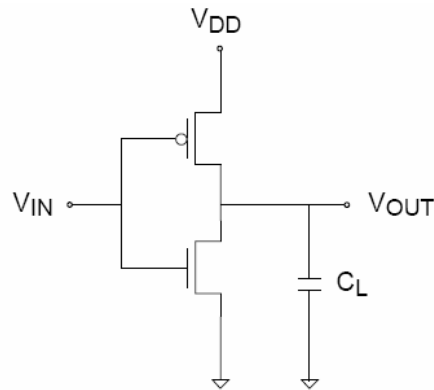
Estimation of important parameters for CMOS inverter: V_M , $A_v(V_M)$, NM_L , NM_H



$$A_v = -(g_{mn} + g_{mp})(r_{on} // r_{op})$$

Lect. 20: Inverters

Estimation of important parameters for CMOS inverter: V_M , $A_v(V_M)$, NM_L , NM_H



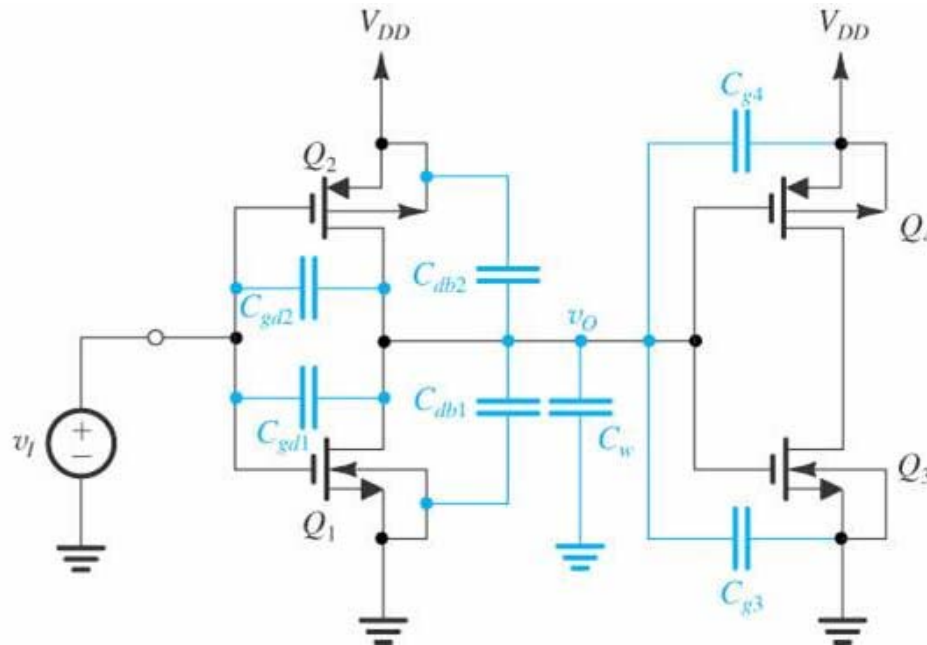
Noise Margins:

$$N_{ML} = V_{IL} = V_M - \frac{V_{DD} - V_M}{|A_v|}$$

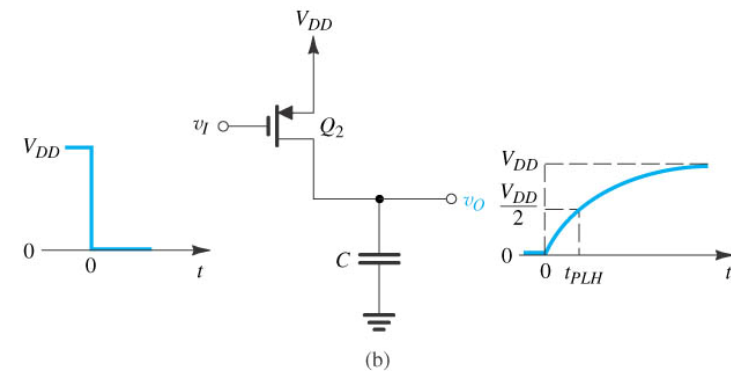
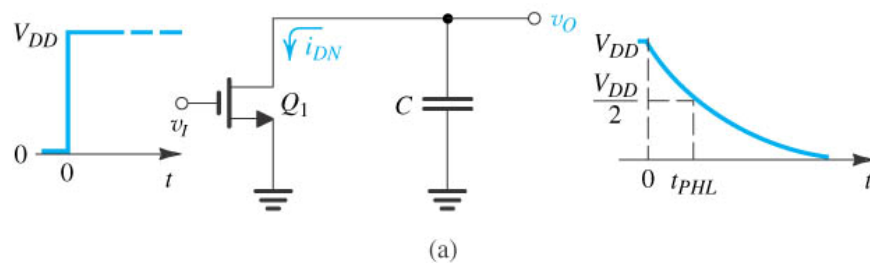
$$N_{MH} = V_{DD} - V_{IH} = V_{DD} - V_M \left(1 + \frac{1}{|A_v|}\right)$$

Lect. 20: Inverters

Estimation of propagation delay: Consider two inverters in a row

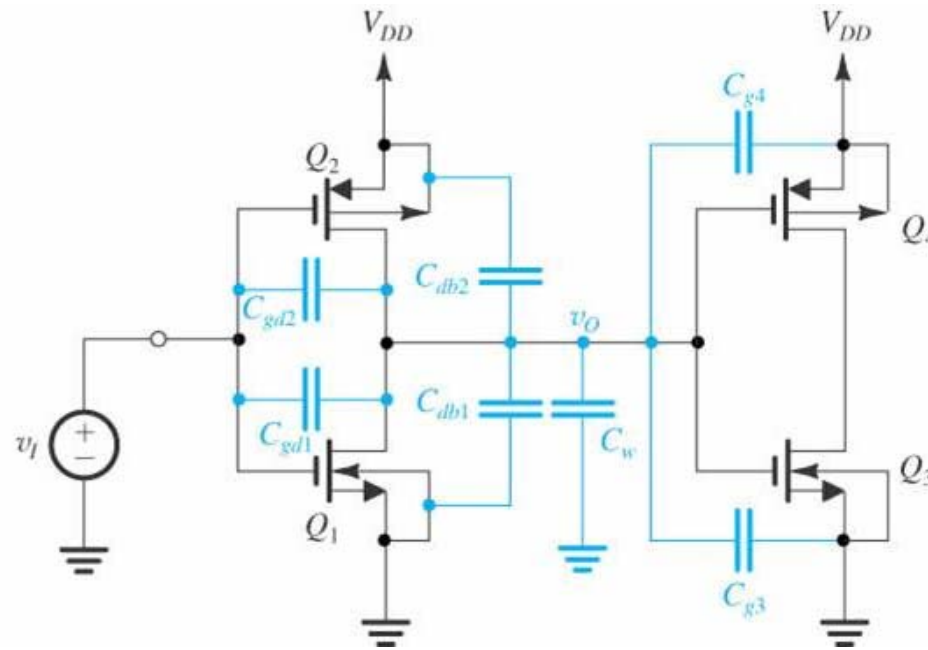


Model with an equivalent capacitance C Between v_O and ground



Lect. 20: Inverters

Estimation of propagation delay: Consider two inverters in a row

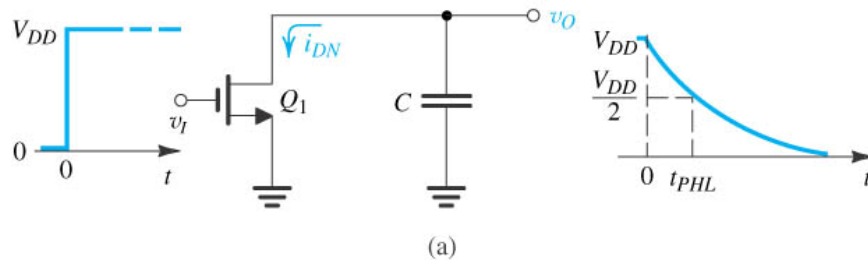


$$C = 2C_{gd1} + 2C_{gd2} + C_{db1} + C_{db2} + C_{g3} + C_{g4} + C_w$$

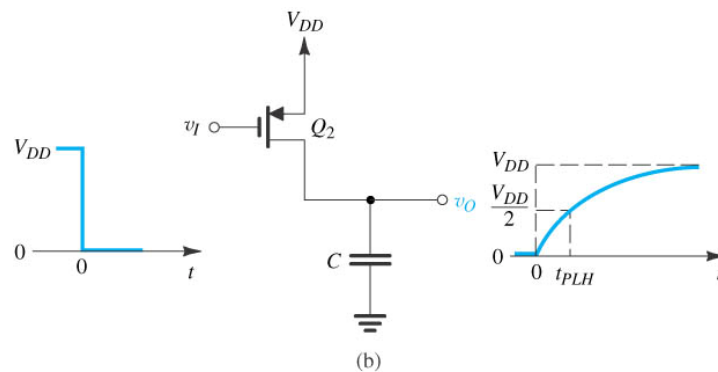
Factor of two for C_{gd} since voltage on both plates changes in the opposite direction

Lect. 20: Inverters

Estimation of propagation delay



$$t_{PHL} \approx \frac{1.7C}{k'_n \left(\frac{W}{L}\right)_n V_{DD}}$$



$$t_{PHL} \approx \frac{1.7C}{k'_p \left(\frac{W}{L}\right)_p V_{DD}}$$

$$t_P = \frac{1}{2}(t_{PHL} + t_{PLH})$$

t_p goes down as V_{DD} goes up and L goes down

Each cycle $Q = CV_{DD}$ is charged and discharged over V_{DD}

Dynamic power consumption: fCV_{DD}^2

→ Make them smaller, smaller, smaller, smaller, smaller, smaller ...